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**14. ABSTRACT**

New control algorithms were developed for robust stabilization of nonlinear dynamical systems. Novel, linear matrix inequality-based synthesis algorithms were developed for the anti-windup problem. Results were extended so that they are applicable to all stabilizable linear systems with input saturation. New insights into the closed-loop behavior of model predictive control were discovered. It was shown that certain model predictive control algorithms induce stability without any robustness. Then it was shown how these algorithms can be modified to guarantee robustness. Formulas for horizon length to guarantee robust stabilization were given, and in the process it was shown that many of the standard assumptions in model predictive control could be relaxed. Extremum seeking control algorithms were advanced, and developed for systems with constraints and with nonsmooth response maps. Finally, some initial progress was made on the use of logical elements in nonlinear control algorithms and the understanding of hybrid control systems in general.

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**FINAL REPORT**

**OPTIMIZATION-BASED ROBUST NONLINEAR CONTROL**

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## Table of Contents

**Summary**

The objective of this research was to further advance optimization-based robust nonlinear control design, for general nonlinear systems (especially in discrete time), for linear systems with input and state constraints, and for systems that need to be positioned at the extremum of an unknown response surface. We accomplished our goals through developments: 1) in the area of anti-windup synthesis, where we provided new optimal synthesis results using linear matrix inequalities, and provided new optimal synthesis results using receding horizon optimal control solutions, 2) in the area of model predictive control for nonlinear systems, establishing stability under greatly weakened assumptions on the optimization problem data, establishing that some existing model predictive control algorithms have no robustness margins, and establishing novel output feedback solutions, and 3) in the area of extremum seeking providing new algorithms to account for constraints and optimization on nonsmooth surfaces. The results of our work are described in thirty journal publications, three book chapters, and forty-eight conference papers.

## Research Publications

The research supported by this grant resulted in 30 published journal papers, 48 refereed conference papers, and 3 book chapters. The published papers are listed below.

Journal papers

1. "Conjugate convex Lyapunov functions for dual linear differential inclusions", R. Goebel, A.R. Teel, T. Hu and Z. Lin, *IEEE Transactions on Automatic Control,* vol. 51, no. 4, pp. 661-666, April 2006.
2. "Exremum seeking methods for optimization of variable cam timing enginer operation", D. Popovic, M. Jankovic, S. Magner, and A.R. Teel, *IEEE Transactions on Control Systems Technology,* vol. 14, no. 3, p. 398-407, May 2006.
3. "A unified framework for input-to-state stability in systems with two time scales", A.R. Teel, L. Moreau and D. Nesic, *IEEE Transactions on Automatic Control,* vol. 48, no. 9, pp. 1526-1544, September 2003.
4. "Anti-windup for stable linear systems with input saturation: an LMI-based synthesis", G. Grimm, J. Hatfield, I. Postlethwaite, A.R. Teel, M. Turner, L. Zaccarian, *IEEE Transactions on Automatic Control,* vol. 48, no. 9, pp. 1509-1525, September 2003.
5. "Nonlinear Scheduled anti-windup design for linear systems", L. Zaccarian and A.R. Teel, *IEEE Transactions on Automatic Control,* vol. 49, no. 11, pp. 2055-2061, Novemeber 2004.
6. "Input-output stability properties of networked control systems", D. Nesic and A.R.Teel, *IEEE Transactions on Automatic Control*, vol. 49, no. 10, pp. 1650-1667, October 2004.
7. "Examples of GES systems that can be driven to infinity by arbitrarily small additive decaying exponentials", A.R. Teel and J. Hespanha, *IEEE Transactions on Automatic Control,* vol. 49, no. 8, pp. 1407-1410, August 2004.
8. "A framework for stabilization of nonlinear sampled-data systems based on their approximate discrete-time models", D. Nesic and A.R. Teel, *IEEE Transactions on Automatic Control*, vol. 49, no. 7, pp. 1103-1122, July 2004.
9. "Further results on robustness of (possibly discontinuous) sample and hold feedback", C.M. Kellett, H. Shim and A. R. Teel, *IEEE Transactions on Automatic Control*, vol. 49, no. 7, pp. 1081-1089, July 2004.
10. "Nonlinear antiwindup applied to Euler-Lagrange Systems", F. Morabito, A.R. Teel and L. Zaccarian, *IEEE Trasnactions on Robotics and Automation,* vol. 20, no. 3, pp. 526—537, June 2004.

11. "Model predictive control: for want of a local control Lyapunov function, all is not lost", G. Grimm, M. Messina, S.E. Tuna, and A.R. Teel, *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 546—558, May 2005.

12. "A nested Matrosov theorem and persistency of excitation for uniform convergence in stable nonautonomous systems", A. Loria, E. Panteley, D. Popovic, and A.R. Teel, *IEEE Transactions on Automatic Control,* vol. 50, no. 2, pp. 183-198, February 2005.

13. "An anti-windup strategy for active vibration isolation systems", A.R. Teel, L. Zaccarian, and J. Marcinkowski, *Control Engineering Practice*, vol. 14, no. 1, pp. 17-27, January 2006.

14. "L2 anti-windup for linear dead-time systems", L. Zaccarian, D. Nesic and A.R. Teel, *Systems & Control Letters*, vol. 54, no. 12, pp. 1205—1217, Decemer 2005.

15. "Conjugate Lyapunov functions for saturated linear systems", T. Hu, R. Goebel, A.R. Teel, and Z. Lin, *Automatica*, vol. 41, no. 11, pp. 1949—1956, November 2005.

16. "Lyapunov characterizations of forced oscillations", T. Hu, A.R. Teel, and Z. Lin, *Automatica*, vol. 41, no. 10, pp. 1723-1735, October 2005.

17. "The L2 (l2) bumpless transfer problem for linear plants: its definition and solution", L. Zaccarian and A.R. Teel, *Automatica*, vol. 41, no. 7, pp. 1273-1280, July 2005.

18. "Discrete-time certainty equivalence output feedback: allowing discontinuous control laws including those from model predictive control", M. Messina, S.E. Tuna, and A.R. Teel, *Automatica*, vol. 41, no. 4, pp. 617--628, April 2005.

19. "Input-to-state stability of networked control systems", D. Nesic and A.R. Teel, *Automatica*, vol. 40, no. 12, pp. 2121-2128, December 2004.

20. "Linear LMI-based external anti-windup augmentation for stable linear systems", G. Grimm, A.R. Teel and L. Zaccarian, *Automatica*, vol. 40, no. 11, pp. 1987--1996, November 2004.

21. "Examples when nonlinear model predictive control is nonrobust", G. Grimm, M. Messina, S.E. Tuna, and A.R. Teel, *Automatica*, vol. 40, no. 10, pp. 1729--1738, October 2004.

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23. "Smooth Lyapunov functions and robustness of stability for difference inclusions", C.M. Kellett and A.R. Teel, *Systems & Control Letters*, vol. 52, no. 2, pp. 395-405, August 2004.

24. "Matrosov theorem for parameterized families of discrete-time systems", D. Nesic and A.R. Teel, *Automatica*, vol. 40, no. 6, pp. 1025-1034, June 2004.

25. "Anti-windup synthesis via sampled-data piecewise affine optimal control", A. Bemporad, A.R. Teel, and L. Zaccarian, *Automatica*, vol. 40, no. 4, pp. 549—562, April 2004.

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1. "Shorter horizons for model predictive control", S.E. Tuna, M. Messina and A.R. Teel, *Proceedings of the 2006 American Control Conference,* pp. 863--868, Minneapolis, MN, June 2006.

2. "Control of saturated linear plants via output feedback containing an internal deadzone loop", *Proceedings of the 2006 American Control Conference,* pp. 5239--5244, Minneapolis, MN, June 2006.

3. "Experimental study of nonlinear anti-windup applied to an integrating linear plant", D. Orlando, L. Zaccarian, and A.R. Teel, *Proceedings of the 2006 American Control Conference,* pp. 5225--5226, Minneapolis, MN, June 2006.

4. "On input-to-state stability of impulsive systems", J.P. Hespanha, D. Liberzon, and A.R. Teel, *Proc. 44$^{th}$ IEEE Conference on Decision and Control,* pp. 4891-4896, Seville, Spain, December 2005.

5. "Regulating discrete-time homogeneous systems under arbitrary switching", S.E. Tuna and A.R. Teel, *Proc. 44$^{th}$ IEEE Conference on Decision and Control,* pp. 2586--2591, Seville, Spain, December 2005.

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7. "Results on input-to-state stability for hybrid systems", C. Cai and A.R. Teel, *Proc. 44$^{th}$ IEEE Conference on Decision and Control,* pp. 5403--5408, Seville, Spain, December 2005.

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11. "Input-output stability of wireless networked control systems", M. Tabbara, D. Nesic, and A.R. Teel, *Proc. 44th IEEE Conference on Decision and Control,* pp. 209--214, Seville, Spain, December 2005.
12. "Regional anti-windup compensation for linear systems with input saturation", T. Hu, A.R. Teel, and L. Zaccarian, *Proceedings of the 2005 American Control Conference*, pp. 3397--3402, Portland, OR, June 2005.
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19. "The L2 (l2) bumpless transfer problem: its definition and solution", L. Zaccarian and A.R. Teel, *Proceedings of the 43rd IEEE Conference on Decision and Control*, pp. 5505—5510, Bahamas, December 2004.
20. "Stability regions for saturated linear systems via conjugate Lyapunov functions", T. Hu, Z. Lin, R. Goebel, and A.R. Teel, *Proceedings of the 43rd IEEE Conference on Decision and Control*, pp. 5499—5504, Bahamas, December 2004.

21. "Generalized dilations and numerically solving discrete-time homogeneous optimization problems", S.E. Tuna and A.R. Teel, *Proceedings of the $43^{rd}$ IEEE Conference on Decision and Control*, pp. 5403--5408, Bahamas, December 2004.

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30. "Robust stabilization of discrete-time nonlinear systems by certainty equivalence output feedback with applications to model predictive control", M. Messina, S.E. Tuna, A.R. Teel, *Proceedings of the 2004 American Control Conference*, pp. 2202--2207, Boston, MA, June/July 2004.

31. "Maneuvering dynamical systems by sliding-mode control", R. Skjetne and A.R. Teel, *Proceedings of the 2004 American Control Conference*, pp. 1277--1282, Boston, MA, June/July 2004.

32. "Results on discrete-time control-Lyapunov functions", C.M. Kellett and A.R. Teel, *Proceedings of the 2004 American Control Conference*, pp. 5961--5966, Boston, MA, June/July 2004.

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40. "Uniform parametric convergence in the adaptive control of manipulators: a case restudied", A. Loria, R. Kelly, and A.R. Teel, *Proceedings of the 2003 IEEE International Conference on Robotics and Automation*, pp. 1062—1067, Taipei, Taiwan, September 2003.

41. "Extremum seeking methods for optimization of variable cam timing engine operation", D. Popovic, M. Jankovic, S. Magner and A. Teel, *Proceedings of the 2003 American Control Conference*, pp. 3136--3141, Denver, CO, June 2003.

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44. "An improved SPSA algorithm for stochastic optimization with bound constraints", D. Popovic, A.R. Teel, and M. Jankovic, *Proceedings of the 16th IFAC World Congress*, Prague, CZ, July 2005 (CD-ROM)

45. "Output feedback compensators for weakened anti-windup of additively perturbed systems", S. Galeani, A.R. Teel and L. Zaccarian, *Proceedings of the 16th IFAC World Congress*, Prague, CZ, July 2005 (CD-ROM)

46. "Stability properties of reset systems", L. Zaccarian, D. Nesic, and A.R. Teel, *Proceedings of the 16th IFAC World Congress*, Prague, CZ, July 2005 (CD-ROM)

47. "Generalized dilations and homogeneity", S.E. Tuna and A.R. Teel, *Proceedings of the 16th IFAC World Congress*, Prague, CZ, July 2005 (CD-ROM)

48. "Hybrid MPC: open-minded but not easily swayed", S.E. Tuna, R.G. Sanfelice, M.J. Messina, and A.R. Teel, *Proceedings of the International Workshop on Assessment and Future Directions of Nonlinear Model Predictive Control*, Freudenstadt-Lauterbad, Germany, August 2005, pp. 169-180.

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**Research Accomplishments**

Anti-windup synthesis

The anti-windup problem is a control synthesis problem in the presence of input saturation. Its distinguishing feature is that a controller has already been designed and it must operate without modification unless the actuators saturate. In this case, because the performance of this controller might be poor in the presence of saturation, anti-windup augmentation should activate to maintain stability and reasonable performance, and to restore closed-loop operation to the original controller when this becomes possible. Systematic solutions to this industry-motivated problem (including the motivation to eliminate pilot/flight control system-induced oscillations) have started to appear over the last decade, and in the course of this grant we contributed general, systematic solutions of various kinds.

*Receding horizon optimal control-based synthesis:* We demonstrated the high performance capabilities of discrete-time disturbance rejection model predictive control in the anti-windup problem. We exploited its efficient off-line synthesis, and have used it for anti-windup synthesis in closed-loops that use high performance continuous-time controllers that ignore input saturation.

*Euler-Lagrange systems:* While anti-windup is usually conceived for linear control systems with input saturation, we showed under a previous AFOSR grant that it can also be posed and solved for more general nonlinear feedback loops. For the current grant, we applied our earlier ideas, with modifications aimed at improved performance, to Euler-Lagrange systems including robotic manipulators such as the SCARA and PUMA. We demonstrated the superior performance of these anti-windup algorithms compared to standard industrial practice which involves path planning to avoid input saturation or ad-hoc anti-windup strategies.

*Systems with time delays:* We presented new results on anti-windup for control systems with time-delays. These results have the potential for significant applications in networked control systems and control of networks.

*Regional anti-windup performance:* Our initial LMI-based anti-windup synthesis algorithms only applied to closed-loop systems having plants that are open-loop exponentially stable. However, by the end of the grant period, we had developed LMI-based synthesis algorithms that were applicable for any closed-loop system. In addition to synthesis algorithms, we provided new algorithms for the analysis of existing anti-windup compensation. Our analysis algorithms, which permit the use of nonquadratic Lyapunov functions, are not convex in general but our experience suggests that the algorithms remain numerically tractable for problems of reasonable size. With this increased applicability, anti-windup algorithms had matured to the state where they are ready to be summarized in a textbook. We have nearly completed such an effort in the course of this grant.

*Robustness, and modeling as differential inclusions:* We continued to investigate the robustness of anti-windup control algorithms and elucidated some natural tradeoffs between performance and robustness in the anti-windup problem that arise from the mere definition of the anti-windup problem. Moreover, we provided some relaxed problem definitions that allow stronger robustness properties. Finally, since input saturation is often modeled as a general sector constraint leading to a linear differential inclusion, we have developed a duality theory for linear differential inclusions, relying on the convex conjugate of a Lyapunov function to provide a Lyapunov function for the linear differential inclusion's dual. This result has significance for numerical investigations of stability for closed-loops with input saturation, since some numerical algorithms (most that are based on nonquadratic Lyapunov functions) give very different answers for stability ranges when applied to the primal problem compared to results when applied to the dual. In addition to these results, we developed a Lyapunov-based frequency response theory for linear differential inclusions.

*Bumpless transfer:* A problem closely related to anti-windup is that of bumpless transfer. The objective of this problem is to provide control logic that enables a smooth transition from one controller (perhaps manual control) to another controller. In the course of the grant, we gave the first formal definition of this problem and provided a systematic solution.

## Model predictive control

Model predictive control (MPC) is an increasingly popular approach used to synthesize stabilizing feedback control laws for nonlinear systems. In industry, this control method typically is based on discrete-time models, which makes determining the MPC feedback law equivalent to solving a finite dimensional optimization problem. The latter often can be carried out on-line, especially for slow plants like those in the chemical process industry. Even for faster plants, including aero-vehicles, MPC is beginning to be considered as a reasonable feedback control alternative.

One of the main challenges in nonlinear MPC is making sure that the finite dimensional optimization problem leads to a stabilizing feedback. Significant attention has been given to this problem over the last fifteen years. The typical solution involves imposing strict terminal constraints and/or limiting the functions used in the optimization problem. One issue that has received very little attention is the nominal robustness of these MPC schemes. In the course of this grant, we showed that some of the standard algorithms are capable of producing closed-loop asymptotic stability with absolutely no robustness. The lack of robustness enters due to state constraints coupled with the use of short horizons when the state constraints are terminal constraints used to induce stability. In all cases where the closed-loop has no robustness, the MPC feedback law is discontinuous, as is the Lyapunov function used to establish asymptotic stability. In our work on this grant, we have showed how to distinguish robustness from zero robustness in discontinuous discrete-time systems. Equipped with these tools, we set out to reformulate MPC algorithms so that robustness is guaranteed. In the process, we aimed to provide as much flexibility as possible in the functions that can be used in the optimization problem.

In our first contribution, we addressed unconstrained MPC and established new results relaxing many of the typical requirements on the functions used in the MPC optimization problem. These relaxations came at the price of requiring sufficiently long horizons, and hence sufficiently large dimension, in the MPC optimization problem. However, we demonstrated constructively on interesting examples that these sufficiently long horizons might actually be reasonably short. These results were enabled by careful exploitation of various aspects of nonlinear systems theory including homogeneity theory.

In our second significant contribution, we addressed constrained MPC extending our earlier results to the problem with inherent state constraints and showing such MPC algorithms can be configured to guarantee nominal robustness. In the process, this work showed that the use of terminal constraints does not necessarily lead to the absence of robustness if sufficiently long horizons are used.

We also showed how nonlinear observability properties can be exploited to implement MPC using output feedback. The only results of this type in the literature of which we are aware have made very non-generic assumptions about the Lipschitz continuity of the MPC feedback law. We demonstrated that this condition is not necessary for implementing MPC by output feedback. This result makes it clear that MPC can be readily applied to nonlinear control systems with limited measurements.

We also showed how to use the MPC idea for homogeneous, switching control systems to develop feedback laws that induce stability that is robust to arbitrary switching. We provided a constructive algorithm that is realizable numerically for systems of low order.

Finally, we developed versions of model predictive control that employ logic variables to assist with the task of making robust decisions. We demonstrated the effectiveness of these algorithms in obstacle avoidance problems involving noisy measurements for highly maneuverable air vehicles.

<u>Extremum Seeking</u>

Extremum Seeking (ES) is an iterative optimization process performed in real time on a physical system that has a significant dynamical aspect. The derivatives of the function to be optimized – system performance in steady state – are not available, but the function can be measured, modulo measurement noise and transients. ES controllers, usually built around numerical optimization algorithms, perform optimization by monitoring the system performance, and adjusting the parameters on-line to improve that performance. In our research we studied applicable numerical algorithms, analyzed the dynamical interaction between an optimization algorithm and the optimized dynamical system, and derived some recipes to achieve faster extremum seeking. We worked with Ford Motor Company to accelerate optimization and calibration of automotive engines. We have demonstrated advanced multi-variable extremum seeking methods in performance optimization for a dual-independent variable cam timing engine and have developed an interactive software product that a non-ES specialist can use for this task.

In work during this grant, we considered extremum seeking problems where the optimal operating condition resides on a constraint boundary. In this case, we showed that the naïve application of certain numerical optimization algorithms, such as simultaneous perturbation stochastic approximation (SPSA) with projection, may exhibit especially slow convergence because of the interaction between the projection operator and a gradient approximation. We then showed how to modify the standard SPSA algorithm to remove this effect and restore fast convergence that can be used in extremum seeking. We have also established new convergence properties using direct search algorithms for nonsmooth optimization problems. These results give a firm theoretical foundation for using direct search within an extremum seeking framework.

Analysis of Networked Control Systems

It is common in modern control applications to find control loops that are closed via a serial communication channel that transmits signals from many sensors and actuators in the system, as well as signals from other unrelated users that are connected to a network. Motivation for using this set-up comes from lower cost, ease of maintenance, great flexibility, as well as low weight and volume. The main issue in these "Networked Control Systems" is that the serial communication channel has many "nodes" (sensors and actuators) where only one node can report its value at a time and, hence, access to the channel needs to be scheduled in an appropriate manner for a proper operation of the control system. During this grant, we contributed to the stability analysis of nonlinear networked control systems. Based on the small gain theorem, we provided estimates for the maximum allowable transfer interval (MATI) in networked control systems that are typically multiple orders of magnitude less conservative than previous results existing in the literature. Along the way, we have developed a useful paradigm for the classification of networked control system protocols that is based on Lyapunov stability theory. We expect this characterization of efficient network protocols to lead to the development of new protocols that allow for larger MATIs.

Hybrid systems: control and analysis

Hybrid systems provide an exciting area of research strongly related to nonlinear control. A strong, foundational understanding of hybrid dynamical systems will allow bringing nonlinear control design tools to bear on a much wider class of systems, and will also enable systematically introducing logic-based switching into nonlinear control algorithm design. Researchers have struggled over the last decade to come up with a concise characterization of hybrid systems and their properties. Inspired by earlier work in this area, we began to develop an array of results that we believe will help to make hybrid systems more tractable. For example, we provided new results on convergence properties of solutions to hybrid systems, and related these properties to robust asymptotic stability. We established a general invariance principle in the spirit of LaSalle's invariance principle for differential and difference equations. Also, we proved that asymptotically stable hybrid systems admit smooth Lyapunov functions. We used these results as motivation to provide new Lyapunov proofs for the stability of networked control systems (both wired and wireless) and for control loops with reset elements, like the so-called "Clegg integrator". We investigated the degree to which the well-known "input-to-state" stability theory for differential equations carries over to hybrid systems. There are some subtle differences that appear in the hybrid domain. Nevertheless, we were able to use hybrid control to induce input-to-state stability with respect to measurement noise where this is impossible to do with pure state feedback. We also gave novel conditions for input-to-state stability in impulsive dynamical systems. As an alternative to the sample and hold solutions that guarantee asymptotic stabilization that is robust to measurement noise for general, asymptotically controllable nonlinear systems, we showed how this property can be induced for the same class of systems using logic-based hysteresis switching. This same idea can be used to give a simple, systematic solution to control problems like global swing-up of the inverted pendulum.

These results are poised to enable important new breakthroughs both in the use of hybrid control for nonlinear systems, and also the development of control algorithms for hybrid systems. These are some of the topics that we are considering in our subsequent work, which is also being supported by AFOSR.

**Personnel Supported**

*Faculty*: Dr. Andrew R. Teel (PI);
*Graduate Student Researchers*: Gene Grimm, Dobrivoje Popovic, Diane Dai

**Transitions**

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*Telephone:* (313) 390-8916.

*Theory transitioned:* Exremum seeking in dynamical systems

*Application:* Rapid engine calibration for a variable cam timing engine.

**Plenary Lectures during period**

IFAC Nonlinear Control Systems Design Symposium, Stuttgart, Germany, 2004.
Asian Control Conference, Melbourne, Australia, 2004.

**Lifetime honors/awards**

IEEE Fellow, 2002
AACC Donald P. Eckman Award, 1999
SIAM Control and Systems Theory Prize, 1998